Exponential Growth and Modeling

Is it Really a Small World After All?

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

Students will apply their knowledge of functions and regressions to compare the U.S. population over time to that of other countries. In particular, students will examine the population of the U.S., Rwanda, and two other countries of their choosing. By requiring students to use these two specific countries, students will examine the two scenarios of both growth and decline. Using the website:
http://data.worldbank.org/indicator/SP.POP.TOTL
students will select data values from the years 1983 - 2013. Students will then be asked to make predictions regarding future populations and research factors that influence population growth.

II. UNIT AUTHOR:

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III. COURSE:

Algebra II

IV. CONTENT STRAND:

Functions, Statistics

V. OBJECTIVES:

The student will be able to:

- Use data provided by The World Bank to create scatter plots to model the population growth over the past 30 years for four countries
- Use the regression feature of either a spreadsheet software or the calculator to find a curve of best fit for the population growth of each of the four selected countries.
- Use the correlation coefficient to aid in the selection of the curve of best fit
- Use the regression model to interpret the population for both previous and future years.

VI. REFERENCE/RESOURCE MATERIALS:

- Classroom set of computers with MS Word (at least 1 computer for each pairing of students) and internet access
- Graphing calculator with linking cable, MS Excel or other statistical software such as Fathom or Minitab
- Student handout and rubric

VII. PRIMARY ASSESSMENT STRATEGIES:

Students will create a lab write-up in an MS Word document that incorporates the computations and graphics created with their graphing calculator or in MS Excel. Students will articulate their interpretations of the data sets and their findings regarding the population growth over time. A handout and a rubric which outlines the components of their write-up are included with this lesson template.

Students will complete this assignment with a partner. This will help with any accommodations that are needed for students with special needs.

VIII. EVALUATION CRITERIA:

Students will gather data on four countries using the website: http://data.worldbank.org/indicator/SP.POP.TOTL so that they have at least two data points for each decade's worth of data. Students will then create an integrated lab report with their partner that will include:

- Scatter plots for each of the four countries
- Regression models for each country
- Analysis of any dips in population growth and research of potential causes for this decline
- Comparison of the model's suggested population with known population values from previous years
- Projections of future population values

IX. INSTRUCTIONAL TIME:

One and a half of a 90 minute period [total 120 minutes]

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Strand

Algebra II, Functions, Statistics

Mathematical Objective(s)

Students will demonstrate mastery of their knowledge of function models and regressions as they assess four sets of population data from The World Bank. More specifically, students will:

- Use data provided by The World Bank to create scatterplots to model the population growth over the past 30 years for four countries
- Use the regression feature of either a spreadsheet software or the calculator to find a curve of best fit for the population growth of each of the four selected countries.
- Use the correlation coefficient to aid in the selection of the curve of best fit
- Use the regression model to interpret the population for previous years and predict the population for future years.

Furthermore, students will demonstrate proficiency with the use of statistical software or their graphing calculator to facilitate the organization of and analysis of data as they use MS Excel and MS Word for creating the graphics and text for their report.

Related SOL

- AII.7 (Functions)
- AII.9 (Statistics)
- AFDA/3 (Modeling Data)

NCTM Standards

- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- Draw reasonable conclusions about a situation being modeled.
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning:

World Geography 3c (Analyzing culture)

Materials/Resources

- Classroom set of graphing calculators with linking cables or statistical software
- Classroom set of computers with internet access

Assumption of Prior Knowledge

- *Prior mathematical knowledge assumed by the task/activity.*

Students should have a functional level of knowledge regarding methods for creating scatter plots, finding regression models, and using the correlation coefficient to determine which curve would best model the data. Students should be familiar with the input/output relationship with functions so that they can make predictions and inferences regarding previous and future y-values for given inputs.

- *Student background knowledge*

The activity that students will be doing in this lesson involves students performing procedures with connections and also "doing" the mathematics. Therefore, students should be proficient with identifying function families, inputting data into statistical software or in their calculator, finding regression models, and creating and using scatter plots to model data.

- *Common language, ideas, and terminologies used*

Students should be using terminology such as scatter plot, regression, correlation coefficient, linear function, quadratic function, cubic function, and exponential function.

- *Possible misconceptions or challenges*

Students sometimes have issues entering their data into the appropriate lists or spreadsheets. Students need to be sure to label their graphs and tables to avoid confusion among the data sets.

Students may also have difficulty with recognizing that their focus needs to be on the r^2 value and not the r value when different regression models are examined. This is because most of these models will not be linear and as such, the r^2 value is most appropriate as it describes the strength of the relationship.

- *Relevant concepts have already been (or should have been) explored prior to this task/activity*
    - Methods for creating scatter plots
    - Methods for finding varying regression models
    - Use of the correlation coefficient to determine which curve best fits the data
    - How to use a graphing calculator, MS Excel, or other statistical software to create scatter plots and calculate regressions

- *What relevant contexts (example: analysis & impact of natural disasters; traffic control; social issues) are drawn on in relation to this concept?*

Students will examine what potential causes in history impact population growth. Because students will investigate the population of Rwanda, students will consider the genocide that occurred in the 1990's and see its impact on that particular country's population over time. Depending upon what other nations the students' select, they may encounter other historic factors that impact population over time.

Introduction: Setting Up the Mathematical Task

In this activity, students will consider the population of four different countries over the course of 30 years. Students will create scatter plots and find regressions to model the data. Students will consider potential factors that influence population decline such as war, famine, natural disasters, or other factors. They will compare the regression's generated values to known values of the population and make inferences about populations in future years.

Time outline: 1.5-90 minute periods:

- 10 minutes - The teacher will introduce the activity, ensure that students understand the parameters of the activity and have all resources that they need available to them. The teacher may want to review how to use the calculator or statistical software to find regression models.
- 20 minutes - Students will select two countries of their choosing in addition to the two required countries. They will need to select at least two years for each decade to record the populations from each country. Students should select the same years for all countries.
- 30 minutes - Students will need to create scatter plots for each country's data and identify any declines in the population over the past 30 years. Students should research any country that experienced a decline during the years which the population declined to determine if there were any obvious reasons that contributed to its decline. Students will create regression models for each country's population over time.
- 60 minutes - Students will create a word processed document where they will record their answers to the analysis questions. These questions will require the use of a calculator and the regression models which they found.

Student Exploration

Whole Class Sharing/Discussion

The teacher will facilitate a class discussion prior to data collection to ensure that students understand the parameters of their assignment.

Small Group Work

Students will work with a partner throughout this activity. Students will select the years and two countries of their choosing in addition to the United States and Rwanda to analyze the population of over the past 30 years. Students will create scatter plots and find regression models to fit the data sets. They will make comparisons about the regression model's population value and known population values for previous years and use the model to make predictions for future years.

Student/Teacher Actions:

- Students work with a partner to investigate the population growth of four countries over time. They will use the computer and The World Bank website to gather values for the population of nations over the past 30 years. Students will use statistical software or their graphing calculator to create scatterplots, find regressions, and make inferences about the data sets. They will record their progress and answer analysis questions in a word processed document.

- The teacher should circulate among the groups and address any technology issues and address issues regarding student understanding. Common causes of confusion among students is that they lose track of which countries correspond with which values. Encourage students to label their data sets with meaningful labels. It would be wise for the teacher to have each group check with her after creating all of the necessary scatter plots and tables for this activity. The teacher can informally assess their understanding of the data being displayed and how the regression is helpful for making connections between the inputted year and outputted population value. This will ensure that all groups are making the appropriate inferences with their graphs and regressions.

- Teachers should encourage students to look at the population values carefully to see if students observe a decline in the numeric values of the population as slight changes can be difficult to see in a scatter plot. The teacher should encourage students to conduct some basic research regarding any of the four countries that they study that experienced a decline during the 30 year period. For example, Rwanda experienced a decline due to the genocide that occurred in the 1990's. This adversely affected the population during that era.

Monitoring Student Responses

Students' responses will be monitored through four distinctive methods for this exploration:

1. Students' discussions with their partner or groups as they gather and analyze their data for the four countries.
2. Students' reports where they justify their reasoning for regression models and make inferences about previous and future population values.
3. Classroom discussion that is facilitated by the teacher prior to the data collection process.
4. Teacher discussion with each pairing of students after they create the graphs and tables required for their write-up.

Assessment List and Benchmarks

Student handout and corresponding rubric.

Is it *Really* a Small World After All?

Every culture is unique and faces its own challenges to its survival over time. In this activity, you will investigate how the population of different countries has changed over the past 30 years. More specifically, you will examine how the population of United States, Rwanda, and two other countries of your choosing has changed over time.

Materials:

- Computer with internet access
- Word processing software (such as MS Word)
- Spreadsheet software (such as MS Excel)

Initial tasks:

1.) Go to the website http://data.worldbank.org/indicator/SP.POP.TOTL and examine the list of countries. Select two countries from the available list that are not the United States or Rwanda.
2.) Record the population for each of the four countries so that you have at least two points from each decade. Use the same years for each of your four countries.
3.) Create a scatter plot and try at least three different regressions to model each country's growth.

After completing these tasks, answer the following questions or complete the following tasks in a word processed document.

Further Analysis:

1.) Which functions did you try for modeling the data for each country? Why did you try these three?
2.) Which, if any, of these countries experienced a decline in their population for any segment of time over the past 30 years. What factors might have contributed to their decline? (Hint: you may want to do a little research on the history of these countries to help you).
3.) Which regression models the data best? How do you know?
4.) Create a table to organize each country with the best regression model. **Use these functions to answer the remaining questions.**
5.) How does your regression model's population calculation for the year 2000 compare with the reported population values for the four countries? Were they over or under estimates?
6.) Project the population for each country in the year 2015. Show work to support your answers.
7.) Assuming your prediction is relatively accurate for the year 2015, which country should grow the most between 2013 and 2015? Explain your reasoning.
8.) Do you think your models could be used to predict the population during 2020? During the year 2100? Explain your reasoning.

Is it *Really* a Small World After All?
Rubric

Criteria	Points Earned	Points Possible
Scatter plots [a total of four, 5 points each] • Axes are labeled • Graphs contain a title that describes the data set		**20**
Question 1 • Three functions are listed for each country - 4 points • Explanation is provided for trying each type of model - 6 points		**10**
Question 2 • Population decline is identified correctly - 5 points • Reasonable potential causes for the decline are addressed - 5 points		**10**
Question 3 • The best model is identified for each country - 4 points • Statistical justification is provided - 6 points		**10**
Question 4 • Table containing four regression equations is provided. • The table is easy to interpret		**5**
Question 5 • Population during the year 2000 is calculated correctly for each country - 4 points • Comparisons are made to the actual values of the population during those years. Over and under estimates are identified correctly - 6 points		**10**
Question 6 • Population predictions are made for the year 2015 - 4 points • Supporting work is present - 6 points		**10**

Question 7 • Population growth between the years 2013 and 2015 is calculated correctly - 8 points • Conclusions regarding which country grew the most are supported - 2 points		**10**
Question 8 • Conclusion regarding whether or not the model could be used for the years 2020 and 2100 - 4 points • Justification is provided - 6 points		**10**
Writing and Grammar • There are few to no typos present • Proper grammar is used • Appropriate statistical and mathematical terminology is used		**5**
Total		**100**

***Student Sample work

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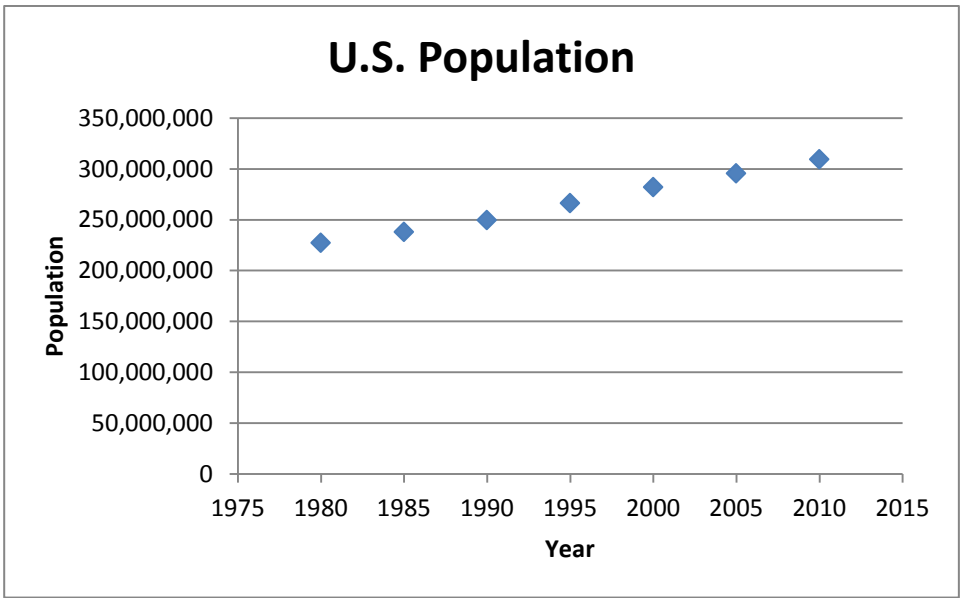
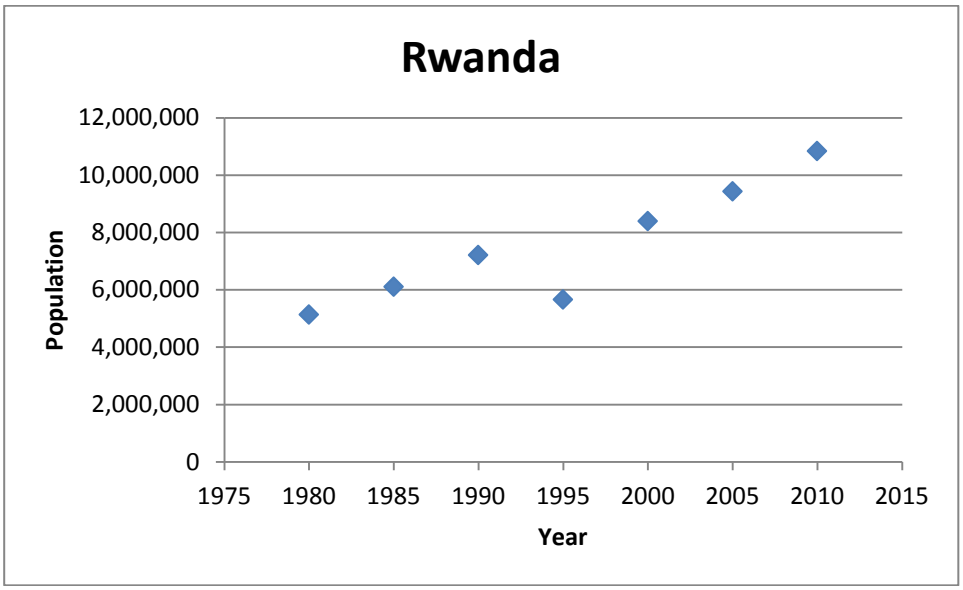
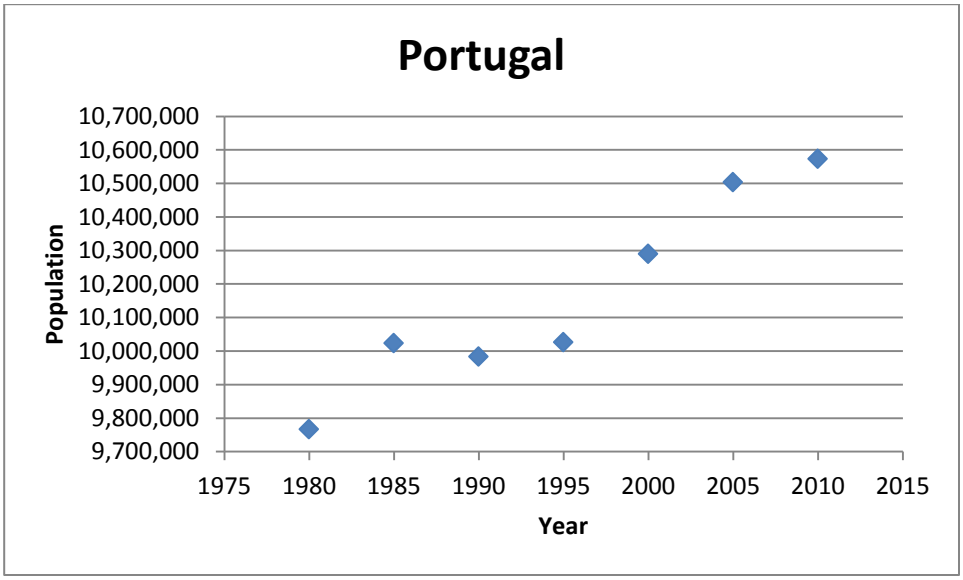
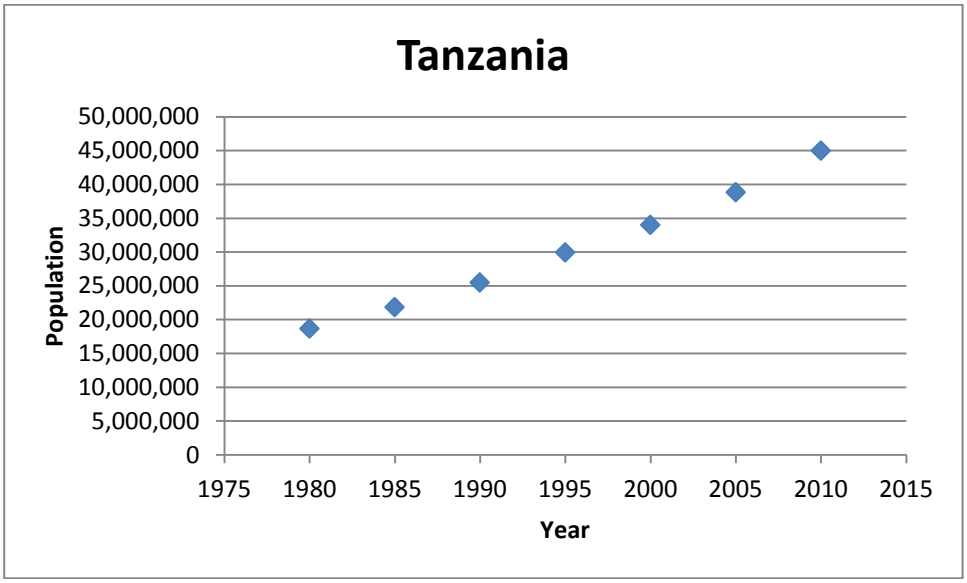
Initial Task:









1.) Which functions did you try for modeling the data for each country? Why did you try these three?

U.S.	ExpReg y=a*b^x a=.1801510996 b=1.010637945 r²=.9973521452 r=.9986751951	LinReg y=ax+b a=2814489.243 b=-5348040996 r²=.9964984892 r=.9982477094	QuadReg y=ax²+bx+c a=10613.44857 b=-39533170.56 c=3.6892688E10 R²=.9975612898
Rwanda	ExpReg y=a*b^x a=5.177464E-14 b=1.023528084 r²=.8226832714 r=.9070188925	LinReg y=ax+b a=177875.1786 b=-347319032.7 r²=.8329301441 r=.9126500666	CubicReg y=ax³+bx²+cx+d a=266.2582222 b=-1588602.403 c=3159511727 d=-2.094672E12 R²=.8903663759

Portugal	ExpReg y=a*b^x a=58541.9902 b=1.002588179 r²=.9202829405 r=.9593137862	LinReg y=ax+b a=26331.98571 b=-42365790.5 r²=.9194970911 r=.9589041094	CubicReg y=ax³+bx²+cx+d a=4.531333333 b=-26751.93095 c=52661283.65 d=-3.455524E10 R²=.9331047716
Tanzania	ExpReg y=a*b^x a=1.811645E-18 b=1.029522725 r²=.9985494528 r=.9992744632	LinReg y=ax+b a=866745.1857 b=-1698616074 r²=.9903585848 r=.9951676164	QuadReg y=ax²+bx+c a=9527.382857 b=-37147512.41 c=3.6219653E10 R²=.9993332466

Because each graph showed a steady increase, a linear fit was initially tried. However, the values shown in each graph changed quickly so an exponential model was also tried. Because the graphs had a curved pattern, polynomials such as cubic and a quadratic were also tried.

2.) Which, if any, of these countries experienced a decline in their population for any segment of time over the past 30 years. What factors might have contributed to their decline? (Hint: you may want to do a little research on the history of these countries to help you).

Both Rwanda and Portugal showed a decline in their populations over this span of 30 years. In the 1990's Rwanda experienced a genocide which causes a significant decline in its population. From the year 1960 - 1990, Portugal experienced a unique decline in its number of young people while its elderly population started to outnumber the number of young individuals, specifically in the northern regions. From this perspective, it's possible that the mortality rate was slightly greater than its infancy rate.

3.) Which regression models the data best? How do you know?
The models that were highlighted with bolded edges in the table earlier are the best models. This was decided based upon the r^2 value. The closer that this value was to one, the stronger the relationship was.

4.) Create a table to organize each country with the best regression model. **Use these functions to answer the remaining questions.**

United States	$f(x) = 0.180 * 1.01^x$
Rwanda	$f(x) = 266.258x^3 - 1,588,602.403x^2 + 3,159,511,727x - (2.094 \times 10^{12})$
Portugal	$f(x) = 4.531x^3 - 26,751.931x^2 + 52,661,283.65x - (3.456 \times 10^{10})$
Tanzania	$f(x) = 9,527.383x^2 - 37,147,512.41x + +(3.622 \times 10^{10})$

5.) How does your regression model's population calculation for the year 2000 compare with the reported population values for the four countries? Were they over or under estimates?

Country	**Model's Value**	**Actual Value**	**Difference**
United States	279,793,019.5	295,516,599	15,723,579.52 (underestimate)
Rwanda	7,860,151.8	8,395,577	535,425.2 (underestimate)
Portugal	10,267,175	10,289,898	22,723 (underestimate)
Tanzania	34,159,744.07	34,020,512	-139232.07 (overestimate)

6.) Project the population for each country in the year 2015. Show work to support your answers.

Country	Work
United States	$f(2015) = 0.180 * 1.01^{2015}$ **= 327,922,242.2**
Rwanda	$f(2015) = 266.258(2015)^3 - 1,588,602.403(2015)^2 + 3,159,511,727(2015) - (2.094 \times 10^{12})$ =**13,783,531.5**
Portugal	$f(2015) = 4.531(2015)^3 - 26,751.931(2015)^2 + 52,661,283.65(2015) - (3.456 \times 10^{10})$ = **10,823,981.42**
Tanzania	$f(2015) = 9,527.383(2015)^2 - 37,147,512.41(2015) + +(3.622 \times 10^{10})$ = **50,733,690.43**

7.) Assuming your prediction is relatively accurate for the year 2015, which country should grow the most between 2013 and 2015? Explain your reasoning.

Country	2013 Value	2015 Value	Growth Amount
United States	316,128,839	**327,922,242.2**	11,793,403.2
Rwanda	11,776,522	**13,783,531.5**	2,007,009.5
Portugal	10,459,806	**10,823,981.42**	364,175.42
Tanzania	49,253,126	**50,733,690.43**	1,480,564.43

As shown in the table, the U.S. grew the most between these years. The next country that grew the most was Rwanda.

8.) Do you think your models could be used to predict the population during 2020? During the year 2100? Explain your reasoning.

The model's could likely be used to approximate the population in 2020 because this year is not too far from the years used to create the regression models. However, if the regression models were used to predict the population in 2100, they would likely be significantly off because this is almost 100 years after those used to create the model. The further one is from the years to create the model, the less accurate the model is. Furthermore, the model cannot take into account other factors which will likely influence populations over the next 100 years such as disease, war, economic circumstances, and social norms.